Chem 440
Spring 2020

This is a practice exam with 2 questions to get you acquainted with the format of our exams. The actual hour exams will have 4 questions and 50 minutes. We recommend that you get your homework and your notes in order, then give yourself 25 minutes to take this practice test open homework, course notes, and your notes.

Hour Exam

Useful numbers and formulas you may need are given at the end.
You must turn in (or upload PDF or JPEG on Moodle if remote) your answer by 11:55 to get full credit.

1. (18 pts) Let's calculate an average. Carbon-14, used in radiodating fossils, decays according to the formula $f(t)=e^{-t/5730}$, where t is the time in years and $f(t)$ is the fraction of ^{14}C left over.

a. (5) **What is** the fraction of ^{14}C left over after 11450 years?

Solution: substitute t = 11450 into the formula so

$$f(t) = e^{-\frac{t}{5730}} = e^{-\frac{11450}{5730}} = 0.1356$$

b. (5+2) $f(t)$ is not yet normalized so its integral from t=0 to t=∞ equals one. (The carbon has a 100% chance or probability =1 of decaying sometime between 0 and ∞!). **Integrate** $f(t)$ from 0 to ∞ to obtain the normalization factor N, and then **write down** the probability distribution $P(t)=f(t)/N$.

Solution: The probability distribution is normalized by the factor N, to get the normalization factor, integrate the normalized function from 0 to ∞

$$\int_0^\infty \frac{e^{-\frac{t}{5730}}}{N} dt = 1$$

So

$$\frac{1}{N}\int_0^\infty e^{-\frac{t}{5730}} dt = \frac{5730}{N} = 1$$

$$N = 5730$$

So the probability distribution $P(t) = \frac{f(t)}{5730}$.

c. (5+2) Now **calculate** the average time $\bar{t}$ for the decay reaction by using $\bar{t} = \int_0^\infty t \cdot P(t)dt$ in years. **Why** is it useful to write decay reactions in the form $f(t)=e^{-t/a}$?

Solution: the average time $\bar{t}$ for the decay reaction is given by

$$\bar{t} = \int_0^\infty t \cdot P(t)dt = \frac{1}{5730}\int_0^\infty t \cdot e^{-\frac{t}{5730}} dt$$

Integrate it by parts so the average time is

$$\frac{1}{5730}\int_0^\infty t \cdot e^{-\frac{t}{5730}} dt = \frac{1}{5730}\big(F(\infty) - F(0)\big) = \frac{1}{5730}(0 - 5730^2) = 5730$$

where $F(t) = -5730te^{-\frac{t}{5730}} - 5730^2 e^{-\frac{t}{5730}}$.

If you write the decay reaction as $e^{-t/a}$, a is just equal to the average decay time!

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2. On exoplanet Gliese 581g, the indigenous Glark (a beast with 6 eyes) has 14 conjugated carbon or oxygen atoms (14 π electrons) in its vision pigment 'glarkinal.'

a. (2) If retinal has length L=15 Å with 12 conjugated atoms, **what is** the length of glarkinal?

Solution: the length of glarkinal is calculated by

$$L = \frac{15\text{Å}}{12} \times 14 = 17.5\text{Å}$$

b. (8) **Calculate** the frequency of light absorbed when a HOMO electron of glarkinal is excited to the LUMO by using the particle-in-a-box model.

Solution:

$$E = \frac{h^2 n^2}{8mL^2}$$

The HOMO orbital for filling 14 electrons is n=7 and the LUMO orbital is n=8. The HOMO to LUMO transition goes from orbital #7 to #8. $m \approx 9.11 \times 10^{-31}\ kg$ (electron mass) and L≈ $17.5\text{Å} = 1.75 \times 10^{-9} m$. So the energy of the two orbitals is

$$E_7 = 9.64 \times 10^{-19} J$$
$$E_8 = 1.26 \times 10^{-18} J$$

Thus the energy difference is $\Delta E = h\nu = 1.26 \times 10^{-18} - 9.64 \times 10^{-19}\ J\ =\ 2.98 \times 10^{-19} J$ and $\nu = 4.50 \times 10^{14}\ Hz$. The wavelength $\lambda = \frac{c}{\nu} \approx 667\ nm$ where $c \approx 3 \times 10^8\ m/s$.

c. (2+2) **What color** does the Glark's visual sensitivity peak at? **What is** the color of glarkinal itself?

Solution: Glark's visual sensitivity absorb red light and the color of glarkinal is the complementary color, i.e. green.

Useful information:
1 atomic mass unit = 1.66 x 10^{-27} kg; mass of electron m_e = 9.109 x 10^{-31} kg
Planck's constant $h = 6.626 \times 10^{-34}$ J·s; note that $\hbar = 2\pi h$ is about 6.28 times larger.
1 Å = 0.1 nm = 100 pm; $c \approx 3 \cdot 10^8$ m/s
$\int e^{-x/a}\, dx = -a\, e^{-x/a}$
$(1/a) \cdot \int x \cdot e^{-x/a}\, dx = -(a+x)e^{-x/a}$
$\lim_{x\to\infty} x e^{-x/a} = 0$
Color and wavelength, and complementary color wheel